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GROUP S_3 CORDIAL PRIME LABELING OF WHEEL RELATED GRAPH

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Abstract

Let $G = (V(G), E(G))$ be a graph. Consider the group S_3. For $u \in S_3$, let $o(u)$ denote the order of u in S_3. Let $g: V(G) \to S_3$ be a function defined in such a way that $xy \in E(G) \Leftrightarrow (o(g(x)), o(g(y))) = 1$. Let $n_j(g)$ denote the number of vertices of G having label j under g. Now g is called a group S_3 cordial prime labeling if $|n_i(g) - n_j(g)| \le 1$ for every $i, j \in S_3, i \ne j$. A graph which admits a group S_3 cordial prime labeling is called a group S_3 cordial prime graph. In this paper, we prove that the Helm graph, Flower graph and $SP(W_n)$ are group S_3 cordial prime.

AMS subject classification: 05C78

Keyword: *Cordial labeling, prime labeling, group S3 cordial prime labeling.*

I. INTRODUCTION

Graphs considered here are finite, undirected and simple. Let A be a group. The order of $a \in A$ is the least positive integer n such that $a^n = e$. We denote the order of a by $o(a)$.
Cahit [1] introduced the concept of cordial labeling.

Definition 1.1. *Let $f: V(G) \to \{0,1\}$ be any function. For each edge xy assign the label $|f(x) - f(y)|$. f is called a cordial labeling if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1. Also the number of edges labeled 0 and the number of edges labeled 1 differ by at most 1.*

Entringer introduced the concept of prime labeling which was later studied by Tout et al.[4]

Definition 1.2. *A prime labeling of a graph G of order n is an injective function $f : V \to \{1,2,\ldots.n\}$ such that for every pair of adjacent vertices u and v, $gcd\{f(u), f(v)\} = 1$.*

Motivated by these two definitions, we introduce group S_3 cordial prime labeling of graphs. Terms not defined here are used in the sense of Harary [3] and Gallian [2].

The greatest common divisor of two integers m and n is denoted by (m,n) and m and n are said to be relatively prime if $(m,n) = 1$. For any real number x, we denote by $\lfloor x \rfloor$, the greatest integer smaller than or equal to x and by $\lceil x \rceil$, we mean the smallest integer greater than or equal to x.

A path is an alternating sequence of vertices and edges, $v_1, e_1, v_2, e_2, \ldots, v_{n-1}, e_{n-1}, v_n$ which are distinct, such that e_i is an edge joining v_i and v_{i+1} for $1 \le i \le n-1$. A path on n vertices is denoted by P_n. A path $v_1, e_1, v_2, e_2, \ldots, v_{n-1}, e_{n-1}, v_n, e_n, v_1$ is called a cycle and a cycle on n vertices is denoted by C_n. Given two graphs G and H, $G + H$ is the graph with vertex set $V(G) \cup V(H)$ and edge set $E(G) \cup E(H) \cup \{uv | u \in V(G), v \in V(H)\}$. A wheel W_n is defined as $C_n + K_1$. In a Wheel, a vertex of degree 3 on the cycle is called a *rim vertex*. A vertex which is adjacent to all the rim vertices is called the *central vertex*. The edges with one end incident with a rim vertex and the other incident with the central vertex are called *spokes*. The *Helm* H_n is obtained from a wheel W_n by attaching a pendent edge at each vertex of the cycle C_n. The *Flower graph* is the graph obtained from a Helm graph H_n by joining each pendent vertex to the central vertex of the Helm. The graph $SP(W_n)$ is obtained from the wheel W_n by subdividing each spoke by a vertex.

II. GROUP S_3 CORDIAL PRIME GRAPHS

Definition 2.1. *Let $g: V(G) \to S_3$ be a function defined in such a way that $xy \in E(G) \Leftrightarrow (o(g(x)), o(g(y)) = 1$. Let $n_j(g)$ denote the number of vertices of G having label j under g. Now g is called a group S_3 cordial prime labeling if $|n_i(g) - n_j(g)| \leq 1$ for every $i, j \in S_3, i \neq j$. A graph which admits a group S_3 cordial prime labeling is called a group S_3 cordial prime graph.*

Definition 2.2. Consider the symmetric group S_3. Let the elements of S_3 be $\{e,a,b,c,d,f\}$ where

$$e = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix};\ a = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix};\ b = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

$$c = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix};\ d = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix};\ f = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

Now $o(e) = 1, o(a) = o(b) = o(c) = 2$ and $o(d) = o(f) = 3$.

Example 2.3. A group S_3 cordial prime labeling of two graphs is given in Fig. 1

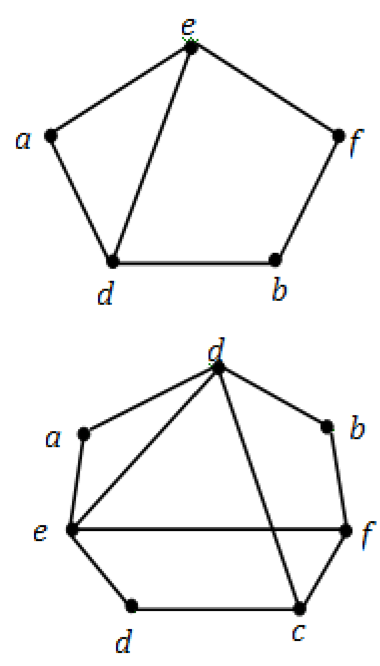

Fig. 1

Definition 2.4. *The Helm H_n is obtained from a wheel W_n by attaching a pendent edge at each vertex of the cycle C_n.*

Theorem 2.5. *Helm graphs H_n are group S_3 cordial prime.*

Proof. Let H_n be the Helm graph. Let w be the center vertex, $u_1, u_2, \ldots, u_n$ be the vertices of the cycle C_n and let $v_1, v_2, \ldots, v_n$ be the pendent vertices.

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Fig.2 shows that H_3 is group S_3 cordial prime.

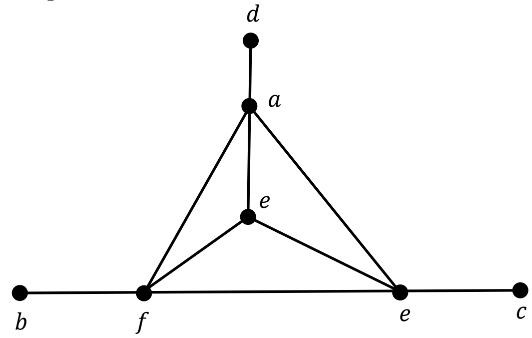

Fig. 2

Case (1): $n \equiv 0(mod\ 6)$.

Let $n = 6k, k \geq 1, k \in Z$.
Define $g: V(H_n) \to S_3$ as follows.

$$g(w) = e$$

$$g(u_i) = \begin{cases} a, & if\ i = 1,7, \dots, 6k-5 \\ d, & if\ i = 2,8, \dots, 6k-4 \\ b, & if\ i = 3,9, \dots, 6k-3 \\ f, & if\ i = 4,10, \dots, 6k-2 \\ c, & if\ i = 5,11, \dots, 6k-1 \\ e, & if\ i = 6,12, \dots, 6k \end{cases}$$

$$g(v_i) = \begin{cases} d, & if\ i = 1,7, \dots, 6k-5 \\ b, & if\ i = 2,8, \dots, 6k-4 \\ f, & if\ i = 3,9, \dots, 6k-3 \\ c, & if\ i = 4,10, \dots, 6k-2 \\ e, & if\ i = 5,11, \dots, 6k-1 \\ a, & if\ i = 6,12, \dots, 6k \end{cases}$$

From Table 1, g is a group S_3 cordial prime labeling.

Case (2): $n \equiv 1(mod\ 6)$.
Let $n = 6k+1, k \geq 1$. Define $g: V(H_n) \to S_3$ as follows:

$$g(w) = e$$

$$g(u_i) = \begin{cases} a, & if\ i = 1,7, \dots, 6k-5 \\ d, & if\ i = 2,8, \dots, 6k-4 \\ b, & if\ i = 3,9, \dots, 6k-3 \\ f, & if\ i = 4,10, \dots, 6k-2 \\ c, & if\ i = 5,11, \dots, 6k-1 \\ e, & if\ i = 6,12, \dots, 6k \\ d, & if\ i = 6k+1 \end{cases}$$

$$g(v_i) = \begin{cases} d, & if\ i = 1,7, \dots, 6k-5 \\ b, & if\ i = 2,8, \dots, 6k-4 \\ f, & if\ i = 3,9, \dots, 6k-3 \\ c, & if\ i = 4,10, \dots, 6k-2 \\ e, & if\ i = 5,11, \dots, 6k-1 \\ a, & if\ i = 6,12, \dots, 6k \\ b, & if\ i = 6k+1 \end{cases}$$

From Table 1, g is a group S_3 cordial prime labeling.

Case (3): $n \equiv 2(mod\ 6)$
Let $n = 6k + 2, k \geq 1$
Define $g: V(H_n) \to S_3$ as follows:

$$g(w) = e$$

$$g(u_i) = \begin{cases} a, & if\ i = 1,7, \dots, 6k-5 \\ d, & if\ i = 2,8, \dots, 6k-4 \\ b, & if\ i = 3,9, \dots, 6k-3 \\ f, & if\ i = 4,10, \dots, 6k-2 \\ c, & if\ i = 5,11, \dots, 6k-1 \\ e, & if\ i = 6,12, \dots, 6k \end{cases}$$

$$g(v_i) = \begin{cases} d, & if\ i = 1,7, \dots, 6k-5 \\ b, & if\ i = 2,8, \dots, 6k-4 \\ f, & if\ i = 3,9, \dots, 6k-3 \\ c, & if\ i = 4,10, \dots, 6k-2 \\ e, & if\ i = 5,11, \dots, 6k-1 \\ a, & if\ i = 6,12, \dots, 6k \\ f, & if\ i = 6k+1 \\ b, & if\ i = 6k+2 \end{cases}$$

From Table1, g is a group S_3 cordial prime labeling.

Case (4): $n \equiv 3(mod\ 6)$
Let $n = 6k + 3, k \geq 1$. Define $g: V(H_n) \to S_3$ as follows:

$$g(w) = e$$

$$g(u_i) = \begin{cases} a, & if\ i = 1,7, \dots, 6k+1 \\ d, & if\ i = 2,8, \dots, 6k+2 \\ b, & if\ i = 3,9, \dots, 6k-3 \\ f, & if\ i = 4,10, \dots, 6k-2 \\ c, & if\ i = 5,11, \dots, 6k-1 \\ e, & if\ i = 6,12, \dots, 6k \\ e, & if\ i = 6k+3 \end{cases}$$

$$g(v_i) = \begin{cases} d, & if\ i = 1,7, \dots, 6k-5 \\ b, & if\ i = 2,8, \dots, 6k-4 \\ f, & if\ i = 3,9, \dots, 6k-3 \\ c, & if\ i = 4,10, \dots, 6k-2 \\ e, & if\ i = 5,11, \dots, 6k-1 \\ a, & if\ i = 6,12, \dots, 6k \\ f, & if\ i = 6k+1 \end{cases}$$

From Table1, g is a group S_3 is a cordial prime labeling.

Case (5): $n \equiv 4(mod\ 6)$
Let $n = 6k + 4$. Define $g: V(H_n) \to S_3$ as follows:

$$g(w) = e$$

For $k \geq 0$, $$g(u_i) = \begin{cases} a, & if\ i = 1,7, \dots, 6k+1 \\ d, & if\ i = 2,8, \dots, 6k+2 \\ b, & if\ i = 3,9, \dots, 6k+3 \\ f, & if\ i = 4,10, \dots, 6k+4 \end{cases}$$

For $k \geq 1$, $$g(u_i) = \begin{cases} c, & if\ i = 5,11, \dots, 6k-1 \\ e, & if\ i = 6,12, \dots, 6k \end{cases}$$

$$g(v_i) = \begin{cases} d, & if\ i = 1,7,\dots,6k+1 \\ b, & if\ i = 2,8,\dots,6k+2 \\ f, & if\ i = 3,9,\dots,6k+3 \\ c, & if\ i = 4,10,\dots,6k+4 \\ e, & if\ i = 5,11,\dots,6k-1 \\ a, & if\ i = 6,12,\dots,6k \end{cases}$$

From Table 1, $g$is a group S_3cordial prime labeling.

Case (6): $n \equiv 5(\bmod 6)$
Let $n = 6k + 5$. Define $g: V(H_n) \to S_3$as follows:

$$g(w) = e$$

$$g(u_i) = \begin{cases} a, & if\ i = 1,7,\dots,6k+1 \\ d, & if\ i = 2,8,\dots,6k+2 \\ b, & if\ i = 3,9,\dots,6k+3 \\ f, & if\ i = 4,10,\dots,6k+4 \\ c, & if\ i = 5,11,\dots,6k-1 \\ e, & if\ i = 6,12,\dots,6k \\ e, & if\ i = 6k+5 \end{cases}$$

$$g(v_i) = \begin{cases} d, & if\ i = 1,7,\dots,6k+1 \\ b, & if\ i = 2,8,\dots,6k+2 \\ f, & if\ i = 3,9,\dots,6k+3 \\ c, & if\ i = 4,10,\dots,6k+4 \\ e, & if\ i = 5,11,\dots,6k-1 \\ a, & if\ i = 6,12,\dots,6k \\ a, & if\ i = 6k+5 \end{cases}$$

From Table 1, g is a group S_3cordial prime labeling.

Table 1

nature of n	$n_a(g)$	$n_b(g)$	$n_c(g)$	$n_d(g)$	$n_e(g)$	$n_f(g)$
$n = 6k + 1$	$2k$	$2k + 1$	$2k$	$2k + 1$	$2k + 1$	$2k$
$n = 6k + 2$	$2k + 1$	$2k + 1$	$2k$	$2k + 1$	$2k + 1$	$2k + 1$
$n = 6k + 3$	$2k + 1$	$2k + 1$	$2k + 1$	$2k + 1$	$2k + 1$	$2k + 2$
$n = 6k + 4$	$2k + 1$	$2k + 2$	$2k + 1$	$2k + 2$	$2k + 1$	$2k + 2$
$n = 6k + 5$	$2k + 2$	$2k + 2$	$2k + 1$	$2k + 2$	$2k + 2$	$2k + 2$
$= 6k$	$2k$	$2k$	$2k$	$2k$	$2k + 1$	$2k$

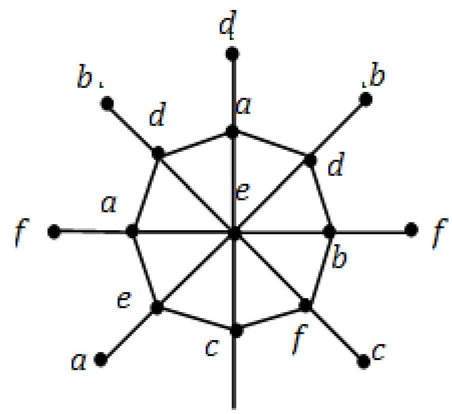

Fig 3

Illustration of the labelings for the Helm graph H_8 is given in Fig 3.

Definition 2.6. *The Flower graph is the graph obtained from a Helm graph H_n by joining each pendent vertex to the central vertex of the Helm.*

Theorem 2.7. *All flower graphs Fl_n are group S_3 cordial prime.*

Proof. Let Fl_n be the flower graph. Let w be the center vertex , let $u_1, u_2, \dots, u_n$ be the vertices of the cycle C_n and let $v_1, v_2, \dots, v_n$ be the pendent vertices of the Helm which are attached to the central vertex w. Number of vertices in $Fl_n = 2n + 1$.
Fl_3 is group S_3 cordial prime from Fig. 4.
Define $g: V(Fl_n) \to S_3$ as in Theorem 2.5.
Clearly g is a group S_3 cordial prime labeling.
Illustration of the labeling for the flower graph Fl_3 is given in Fig. 4.

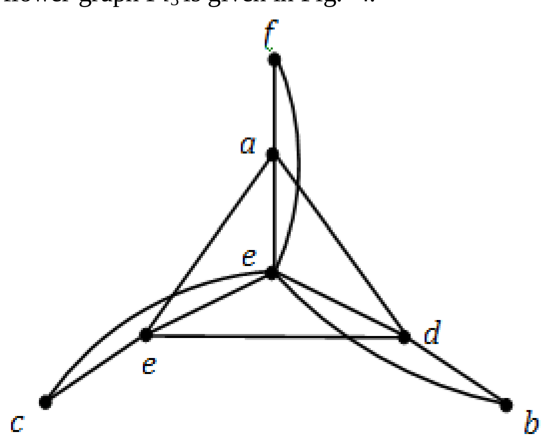

Fig 4

Definition 2.8. *The graph $SP(W_n)$ is obtained from the wheel W_n by subdividing each spoke by a vertex.*

Theorem 2.9. *The graphs $SP(W_n)$ are group S_3 cordial prime.*

Proof. *Let W_n= C_n+ K_1 be the Wheel. Let w be the center vertex and let $u_1,u_2,\ldots,u_n$ be the vertices on the cycle C_n. Let the spokes be subdivided by the vertices $v_1,v_2,\ldots,v_n$.*
$SP(W_3)$ is group S_3 cordial prime from Fig. 6..

Case (1): $n \equiv 0(\bmod 6)$
Define $g: V(SP(W_n)) \rightarrow S_3$ *as follows:*

$$g(w) = e$$

For $k \geq 1$,

$$g(u_i) = \begin{cases} a, & if\ i = 1,7,\ldots,6k-5 \\ d, & if\ i = 2,8,\ldots,6k-4 \\ b, & if\ i = 3,9,\ldots,6k-3 \\ f, & if\ i = 4,10,\ldots,6k-2 \\ c, & if\ i = 5,11,\ldots,6k-1 \\ e, & if\ i = 6,12,\ldots,6k \end{cases}$$

$$g(v_i) = \begin{cases} d, & if\ i = 1,7,\ldots,6k-5 \\ b, & if\ i = 2,8,\ldots,6k-4 \\ f, & if\ i = 3,9,\ldots,6k-3 \\ c, & if\ i = 4,10,\ldots,6k-2 \\ e, & if\ i = 5,11,\ldots,6k-1 \\ a, & if\ i = 6,12,\ldots,6k \end{cases}$$

From Table 2,g is a group S_3cordial prime labeling.

Case (2): $n \equiv 1(mod6)$
Let $n = 6k+1, k \geq 1$
Define $g: V(SP(W_n)) \rightarrow S_3$as follows:

$$g(w) = e$$

$$g(u_i) = \begin{cases} a, & if\ i = 1,7,\ldots,6k-5 \\ d, & if\ i = 2,8,\ldots,6k-4 \\ b, & if\ i = 3,9,\ldots,6k-3 \\ f, & if\ i = 4,10,\ldots,6k-2 \\ c, & if\ i = 5,11,\ldots,6k-1 \\ e, & if\ i = 6,12,\ldots,6k \\ d, & if\ i = 6k+1 \end{cases}$$

$$g(v_i) = \begin{cases} d, & if\ i = 1,7,\ldots,6k-5 \\ b, & if\ i = 2,8,\ldots,6k-4 \\ f, & if\ i = 3,9,\ldots,6k-3 \\ c, & if\ i = 4,10,\ldots,6k-2 \\ e, & if\ i = 5,11,\ldots,6k-1 \\ a, & if\ i = 6,12,\ldots,6k \\ b, & if\ i = 6k+1 \end{cases}$$

From Table 2, g is a group S_3cordial prime labeling.

Case (3): $n \equiv 2(mod\ 6)$
Let $n = 6k+2, k \geq 1$
Define $g: V(SP(W_n)) \rightarrow S_3$*as follows:*

$$g(w) = e$$

For $k \geq 1$

$$g(u_i) = \begin{cases} a, & if\ i = 1,7, \dots ,6k+1 \\ d, & if\ i = 2,8, \dots ,6k+2 \\ b, & if\ i = 3,9, \dots ,6k-3 \\ f, & if\ i = 4,10, \dots ,6k-4 \\ c, & if\ i = 5,11, \dots ,6k-5 \\ e, & if\ i = 6,12, \dots ,6k \end{cases}$$

$$g(v_i) = \begin{cases} d, & if\ i = 1,7, \dots ,6k-5 \\ b, & if\ i = 2,8, \dots ,6k-4 \\ f, & if\ i = 3,9, \dots ,6k-3 \\ c, & if\ i = 4,10, \dots ,6k-2 \\ e, & if\ i = 5,11, \dots ,6k-1 \\ a, & if\ i = 6,12, \dots ,6k \\ f, & if\ i = 6k+1 \\ b, & if\ i = 6k+2 \end{cases}$$

From Table 2, g is a group S_3cordial prime labeling.

Case (4): $n \equiv 3(mod\ 6)$
Let $n = 6k + 3, k \geq 1$
Define $g: V(SP(W_n)) \to S_3$as follows:

$$g(w) = e$$

For$k \geq 1$,

$$g(u_i) = \begin{cases} a, & i = 1,7, \dots ,6k+1 \\ d, & i = 2,8, \dots ,6k+2 \\ b, & i = 3,9, \dots ,6k-3 \\ f, & i = 4,10, \dots ,6k-2 \\ c, & i = 5,11, \dots ,6k-1 \\ e, & i = 6,12, \dots ,6k \\ e, & i = 6k+3 \end{cases}$$

$$g(v_i) = \begin{cases} d, & i = 1,7, \dots ,6k-5 \\ b, & i = 2,8, \dots ,6k-4 \\ f, & i = 3,9, \dots ,6k-3 \\ c, & i = 4,10, \dots ,6k-2 \\ e, & i = 5,11, \dots ,6k-1 \\ a, & i = 6,12, \dots ,6k \\ f, & i = 6k+1 \\ b, & i = 6k+2 \\ c, & i = 6k+3 \end{cases}$$

From Table 2, g is a group S_3cordial prime labeling.

Case (5): $n \equiv 4(mod\ 6)$
Let $n = 6k + 4$
Define $g: V\big(SP(W_n)\big) \to S_3$ *as follows:*

$$g(w) = e$$

$$g(u_i) = \begin{cases} a, & if\ i = 1,7, \dots ,6k+1 \\ d, & if\ i = 2,8, \dots ,6k+2 \\ b, & if\ i = 3,9, \dots ,6k+3 \\ f, & if\ i = 4,10, \dots ,6k+4 \\ c, & if\ i = 5,11, \dots ,6k-1 \\ e, & if\ i = 6,12, \dots ,6k \end{cases}$$

$$g(v_i) = \begin{cases} d, & if\ i = 1,7, \dots, 6k+1 \\ b, & if\ i = 2,8, \dots, 6k+2 \\ f, & if\ i = 3,9, \dots, 6k+3 \\ c, & if\ i = 4,10, \dots, 6k+4 \\ e, & if\ i = 5,11, \dots, 6k-1 \\ a, & if\ i = 6,12, \dots, 6k \end{cases}$$

From Table 2, g is a group S_3cordial prime labeling.

Case (6): $n \equiv 5(mod\ 6)$
Let $n = 6k + 5$
Define $g: V(SP(W_n)) \to S_3$as follows:

$$g(w) = e$$

$$g(u_i) = \begin{cases} a, & i = 1,7, \dots, 6k+1 \\ d, & i = 2,8, \dots, 6k+2 \\ b, & i = 3,9, \dots, 6k+3 \\ f, & i = 4,10, \dots, 6k+4 \\ c, & i = 5,11, \dots, 6k-1 \\ e, & i = 6,12, \dots, 6k \\ e, & i = 6k+5 \end{cases}$$

$$g(v_i) = \begin{cases} d, & if\ i = 1,7, \dots, 6k+1 \\ b, & if\ i = 2,8, \dots, 6k+2 \\ f, & if\ i = 3,9, \dots, 6k+3 \\ c, & if\ i = 4,10, \dots, 6k+4 \\ e, & if\ i = 5,11, \dots, 6k-1 \\ a, & if\ i = 6,12, \dots, 6k \\ a, & if\ i = 6k+5 \end{cases}$$

From Table 2, g is a group S_3cordial prime labeling.

Table 2

$nature\ of\ n$	$n_a(g)$	$n_b(g)$	$n_c(g)$	$n_d(g)$	$n_e(g)$	$n_f(g)$
$n = 6k + 1$	$k + 1$	$k + 2$	$k + 1$	$k + 2$	$k + 2$	$k + 1$
$n = 6k + 2$	$k + 2$	$k + 2$	$k + 1$	$k + 2$	$k + 2$	$k + 2$
$n = 6k + 3$	$k + 2$	$k + 2$	$k + 2$	$k + 2$	$k + 2$	$k + 3$
$n = 6k + 4$	$k + 2$	$k + 3$	$k + 2$	$k + 3$	$k + 2$	$k + 3$
$n = 6k + 5$	$k + 3$	$k + 3$	$k + 2$	$k + 3$	$k + 3$	$k + 3$
$n = 6k$	$k + 3$	$k + 3$	$k + 3$	$k + 3$	$k + 4$	$k + 3$

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Illustration of the labelings for the graph $SP(W_3)$ is given in Fig. 5.

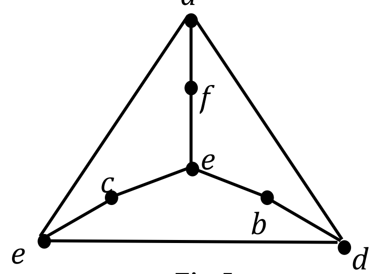

Fig. 5

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